Unimelb Code Masters Bits 2017
Problem B0

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Rules reminder

Teachers please ensure that students have no internet access, and only use language references locally on their computers; not libraries of past problems, how-to-solve-it helpers, and so on.

Mod reminder

The mod operator gives the remainder of a division: $y \mod z$ gives the remainder of y divided by z. For example, $4 \bmod 1 = 0$, $4 \bmod 2 = 0$, $4 \bmod 3 = 1$ and $4 \bmod 4 = 0$. All programming languages and even Excel have a mod operator (usually a %). If you do not know what the mod operator is in your language, you can ask your teacher. (No internet, remember!)

Recursive sequences

A recursive sequence of numbers is one where the nth element of the sequence depends on some previous elements in the sequence. For example, the Fibonacci sequence has the nth element as the sum of the previous two elements.

$$1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

This can be written in maths notation by using subscripts on variables to indicate the element of the sequence. So in the Fibonacci example,

$$x_n = x_{n-1} + x_{n-2},$$

and $x_0 = x_1 = 1$ to get things started.

The Problem

A Linear Congruential Generator is an algorithm that gives a sequence of pseudo-random numbers. Generally, the nth number in the sequence, where $n > 0$, is given by the formula:

$$x_n = (Ax_{n-1} + C) \bmod M,$$

where A, C and M are integers given as parameters of the generator, and x_0 is some integer seed.

An example of the "random" sequence returned when $A = 1$, $C = 6$ and $M = 8$ using a seed of $x_0 = 0$ is

$$0, 6, 4, 2, 0, 6, 4, 2, 0, \ldots$$

Note that this sequence begins repeating after 4 numbers. (Tip: never use these values to make random numbers!)

With different values of A and C, we can get all 8 possible numbers produced (if you mod something by 8, then you can only get the numbers 0 to 7.) For example, $A = 5$ and $C = 23$, you get the sequence

$$0, 7, 2, 1, 4, 3, 6, 5, 0, 7, 2, 1, 4, 3, 6, 5, 0, \ldots$$

Assume $x_0 = 0$ for all three parts of the question.

Question B0, Part 1

Assuming $M = 11$, how many numbers in the range $0, 1, \ldots, 10$ do **not** appear in the generated sequence if $A = 7$, $C = 7$?

Question B0, Part 2

How many distinct/unique numbers do occur in the sequence when $A = 97$, $C = 203$ and $M = 123456$?

Question B0, Part 3

Using $A = 1664525$, $C = 1013904223$ and $M = 2^{32}$, we get a pseudo-random sequence $\{s_0, s_1, \ldots\}$. We will use these numbers to determine a walk on the Cartesian plane using the following rules:

- if $s_i \bmod 5 = 0$ move one step to the right ($x \to x + 1$);
- if $s_i \bmod 5 = 1$ move one step to the left ($x \to x - 1$);
- if $s_i \bmod 5 = 2$ move one step up ($y \to y + 1$);
- if $s_i \bmod 5 = 3$ move one step down ($y \to y - 1$); and
- if $s_i \bmod 5 = 4$ do nothing.

Starting at $(0, 0)$, the sequence of positions will be

$$(0,0),(0,-1),(0,0),(0,1),(0,1),(0,2),(0,3),(0,3),(0,2),(0,1),(0,1),(0,2),(0,2),(0,3),(1,3),\ldots$$

After the first 3 steps we are positioned at $(0, 1)$, after 11 steps at $(0, 2)$ and so on.

If we start at $(0, 0)$, how many steps are required to get to $(44, -56)$?